UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical &
Computer Engineering

ECE 204 *Numerical methods*

Hooke-Jeeves method for finding extrema in *n* dimensions

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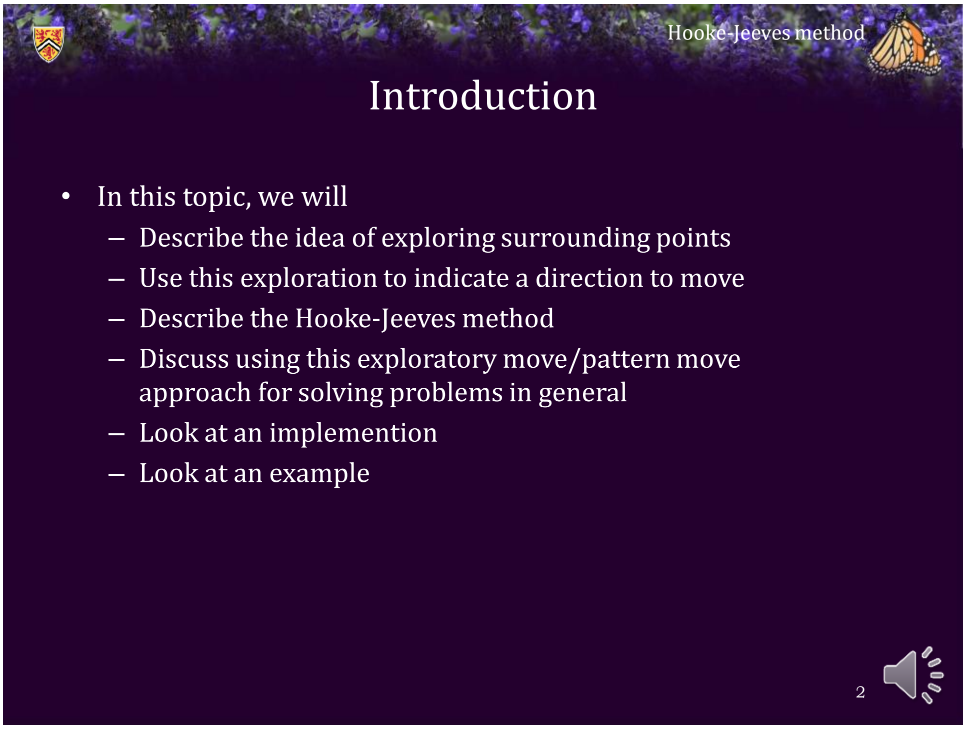

Hooke-Jeeves method

Introduction

- In this topic, we will
  - Describe the idea of exploring surrounding points
  - Use this exploration to indicate a direction to move
  - Describe the Hooke-Jeeves method
  - Discuss using this exploratory move/pattern move approach for solving problems in general
  - Look at an implemention
  - Look at an example

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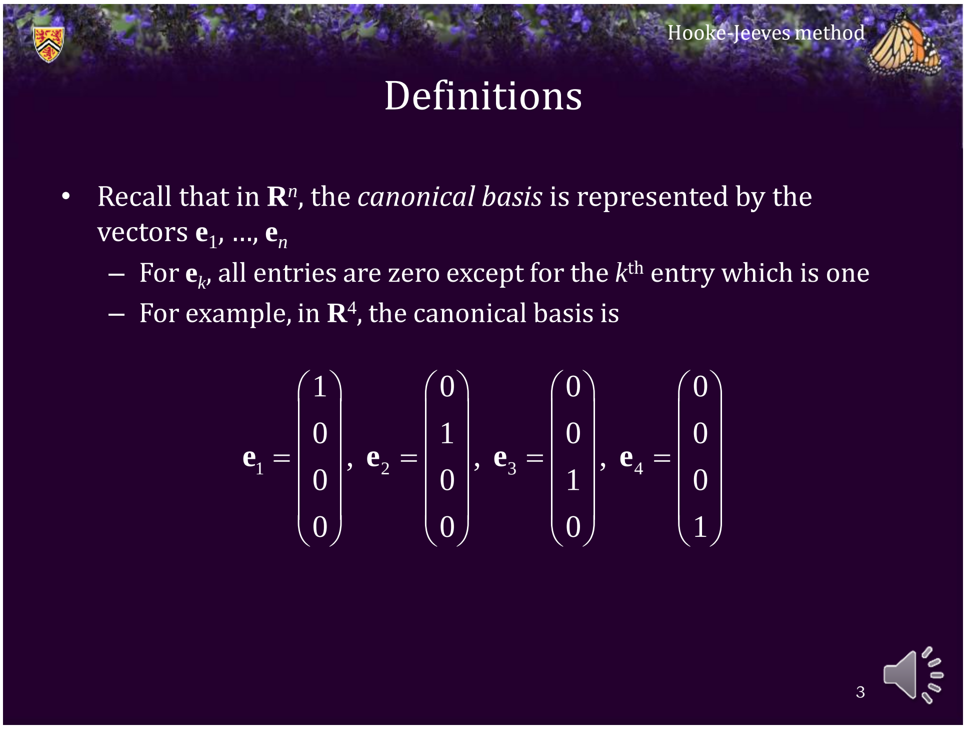

Hooke-Jeeves method

Definitions

- Recall that in $\mathbf{R}^n$, the *canonical basis* is represented by the vectors $\mathbf{e}_1, \ldots, \mathbf{e}_n$
  - For $\mathbf{e}_k$, all entries are zero except for the kth entry which is one
  - For example, in $\mathbf{R}^4$, the canonical basis is

$$\mathbf{e}_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix},\ \mathbf{e}_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix},\ \mathbf{e}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix},\ \mathbf{e}_4 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

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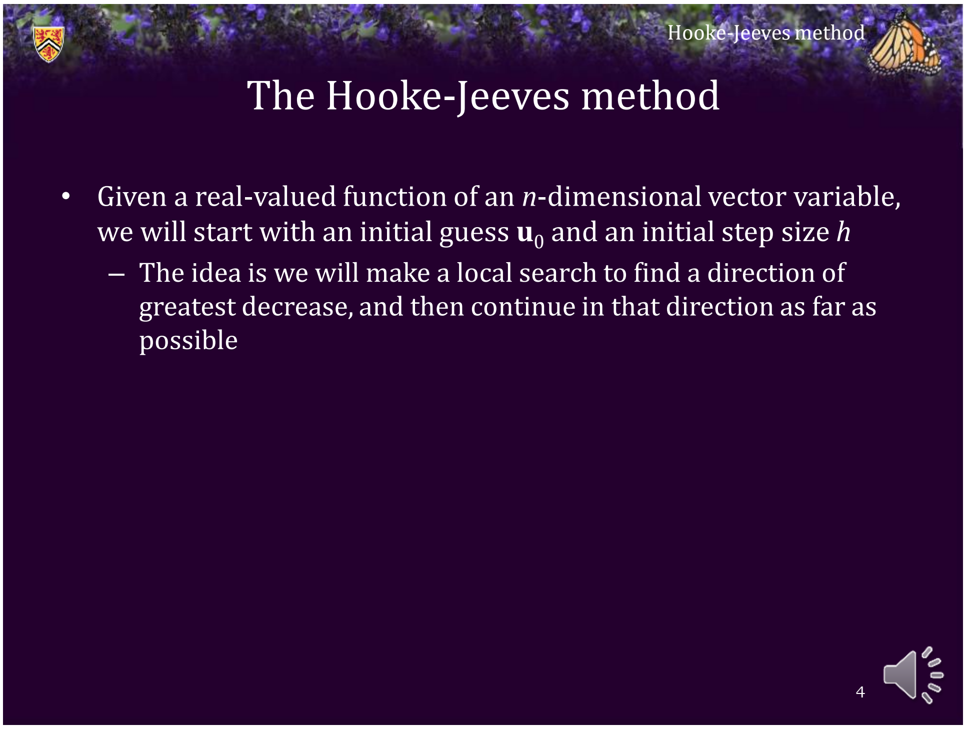

Hooke-Jeeves method

The Hooke-Jeeves method

- Given a real-valued function of an n-dimensional vector variable, we will start with an initial guess $\mathbf{u}_0$ and an initial step size h
  - The idea is we will make a local search to find a direction of greatest decrease, and then continue in that direction as far as possible

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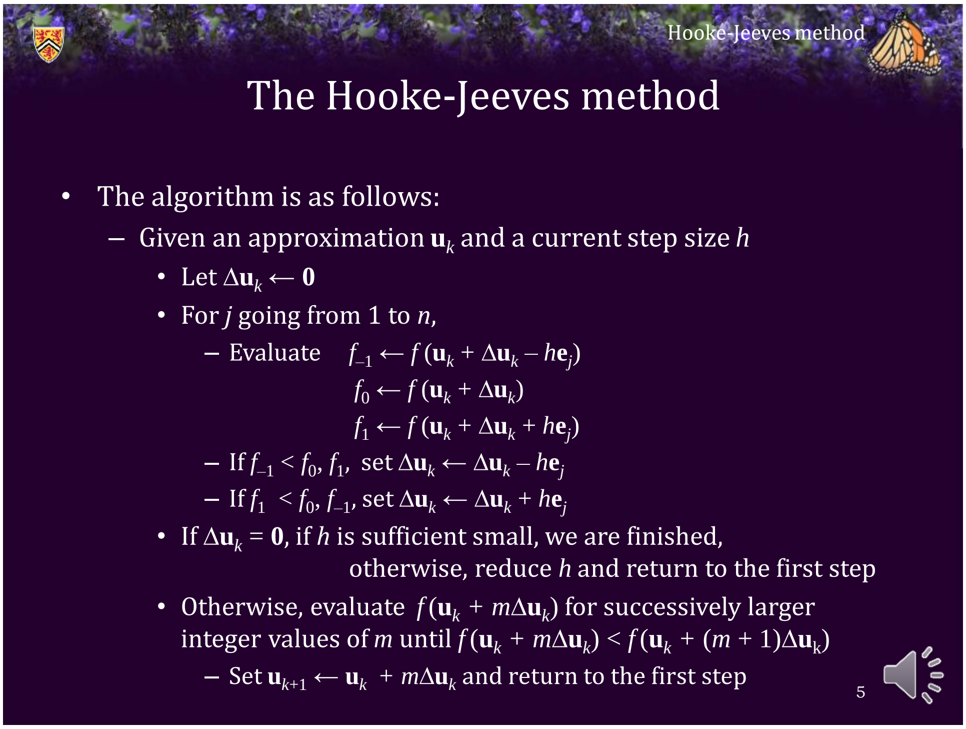
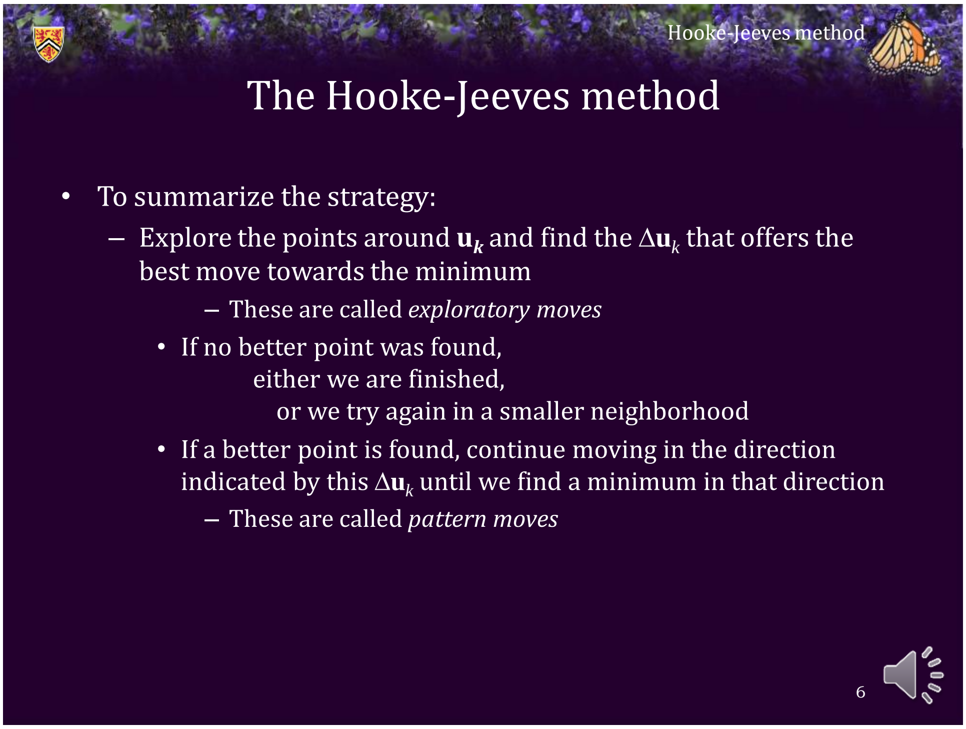


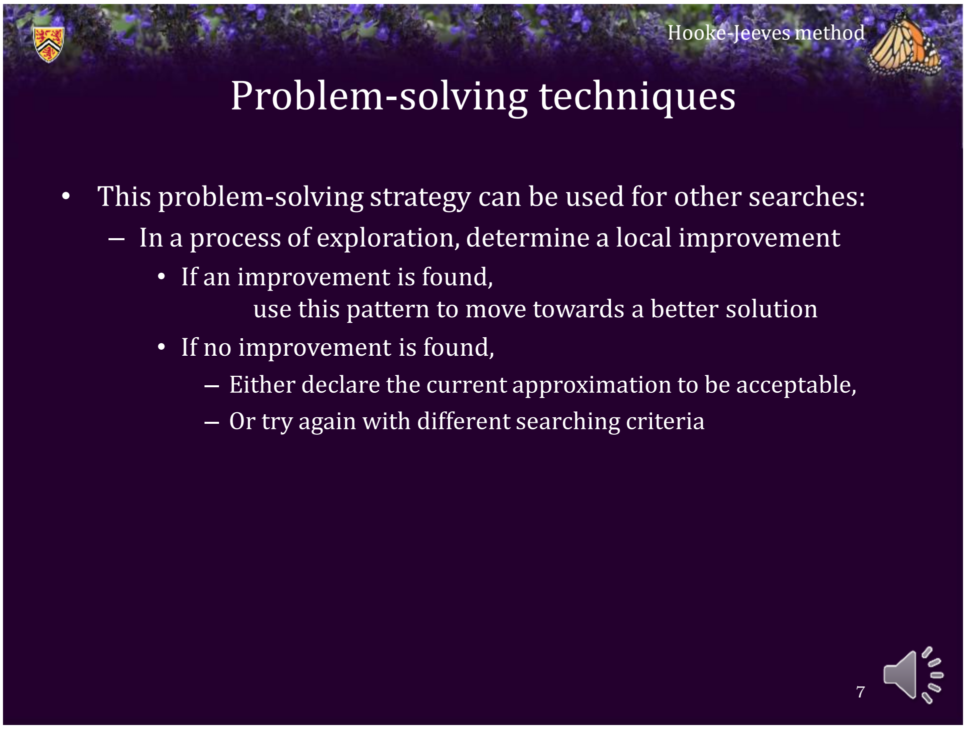
The Hooke-Jeeves method

- The algorithm is as follows:
  - Given an approximation $\mathbf{u}_k$ and a current step size h
    - Let $\Delta\mathbf{u}_k \leftarrow \mathbf{0}$
    - For j going from 1 to n,
      - Evaluate $f_{-1} \leftarrow f(\mathbf{u}_k + \Delta\mathbf{u}_k - h\mathbf{e}_j)$
        $f_0 \leftarrow f(\mathbf{u}_k + \Delta\mathbf{u}_k)$
        $f_1 \leftarrow f(\mathbf{u}_k + \Delta\mathbf{u}_k + h\mathbf{e}_j)$
      - If $f_{-1} < f_0, f_1$, set $\Delta\mathbf{u}_k \leftarrow \Delta\mathbf{u}_k - h\mathbf{e}_j$
      - If $f_1 < f_0, f_{-1}$, set $\Delta\mathbf{u}_k \leftarrow \Delta\mathbf{u}_k + h\mathbf{e}_j$
    - If $\Delta\mathbf{u}_k = \mathbf{0}$, if h is sufficient small, we are finished,
      otherwise, reduce h and return to the first step
    - Otherwise, evaluate $f(\mathbf{u}_k + m\Delta\mathbf{u}_k)$ for successively larger integer values of m until $f(\mathbf{u}_k + m\Delta\mathbf{u}_k) < f(\mathbf{u}_k + (m + 1)\Delta\mathbf{u}_k)$
      - Set $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_k + m\Delta\mathbf{u}_k$ and return to the first step



The Hooke-Jeeves method

- To summarize the strategy:
  - Explore the points around $\mathbf{u}_k$ and find the $\Delta\mathbf{u}_k$ that offers the best move towards the minimum
      - These are called *exploratory moves*
    - If no better point was found,
      either we are finished,
      or we try again in a smaller neighborhood
    - If a better point is found, continue moving in the direction indicated by this $\Delta\mathbf{u}_k$ until we find a minimum in that direction
      - These are called *pattern moves*



Problem-solving techniques

- This problem-solving strategy can be used for other searches:
  - In a process of exploration, determine a local improvement
    - If an improvement is found,
      use this pattern to move towards a better solution
    - If no improvement is found,
      - Either declare the current approximation to be acceptable,
      - Or try again with different searching criteria

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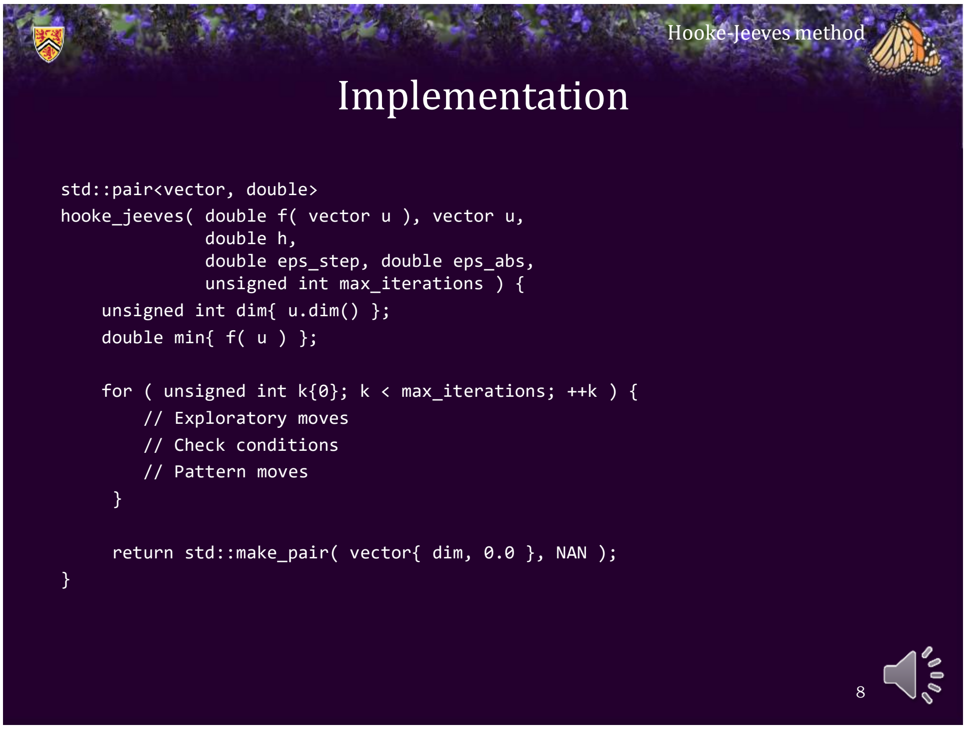

Implementation

```
std::pair<vector, double>
hooke_jeeves( double f( vector u ), vector u,
              double h,
              double eps_step, double eps_abs,
              unsigned int max_iterations ) {
    unsigned int dim{ u.dim() };
    double min{ f( u ) };

    for ( unsigned int k{0}; k < max_iterations; ++k ) {
        // Exploratory moves
        // Check conditions
        // Pattern moves
     }

     return std::make_pair( vector{ dim, 0.0 }, NAN );
}
```

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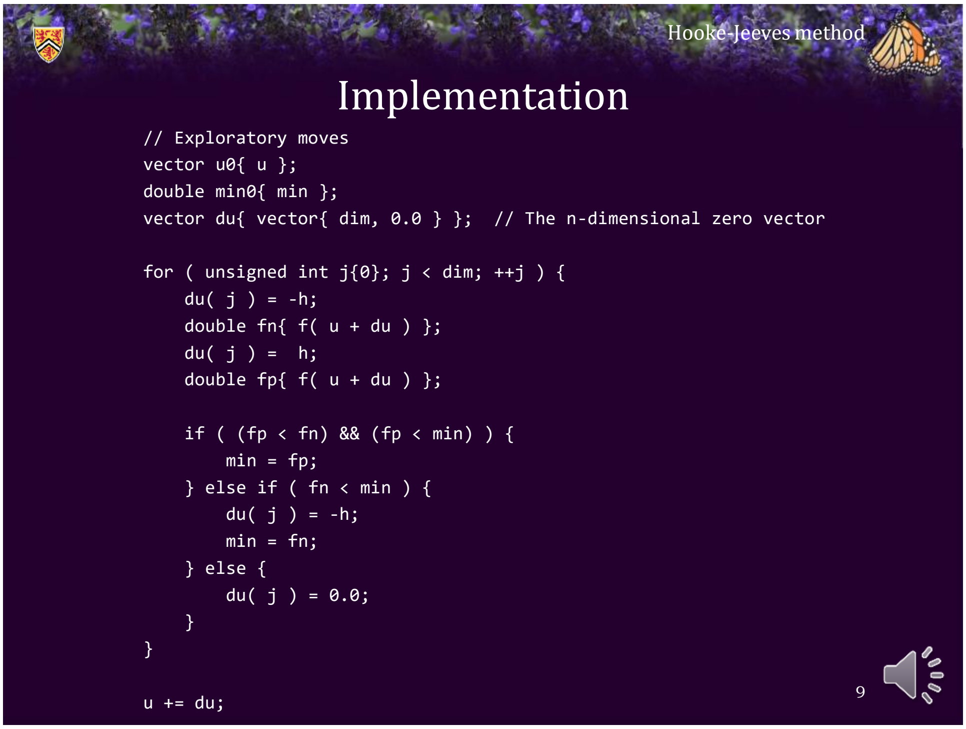
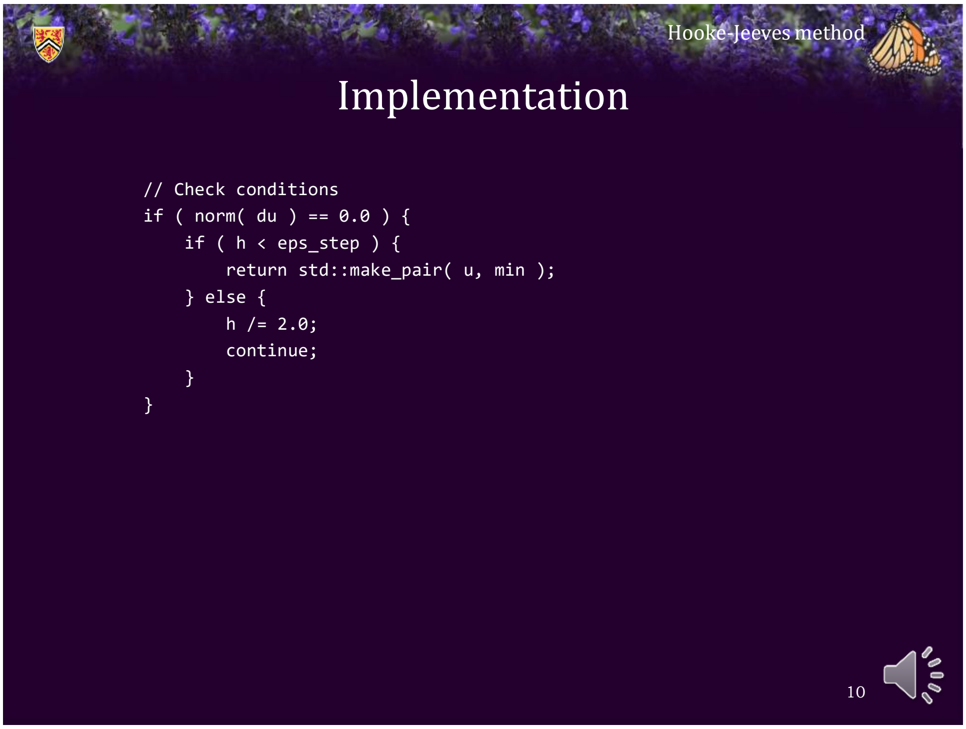


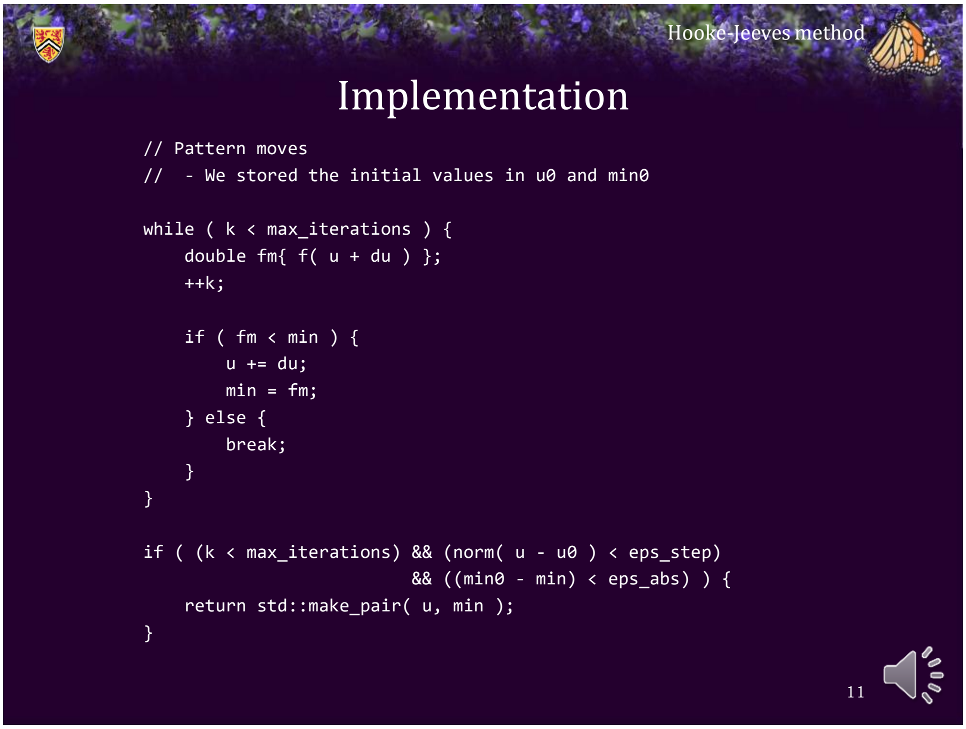
Implementation

```
// Exploratory moves
vector u0{ u };
double min0{ min };
vector du{ vector{ dim, 0.0 } };  // The n-dimensional zero vector

for ( unsigned int j{0}; j < dim; ++j ) {
    du( j ) = -h;
    double fn{ f( u + du ) };
    du( j ) =  h;
    double fp{ f( u + du ) };

    if ( (fp < fn) && (fp < min) ) {
        min = fp;
    } else if ( fn < min ) {
        du( j ) = -h;
        min = fn;
    } else {
        du( j ) = 0.0;
    }
}

u += du;
```



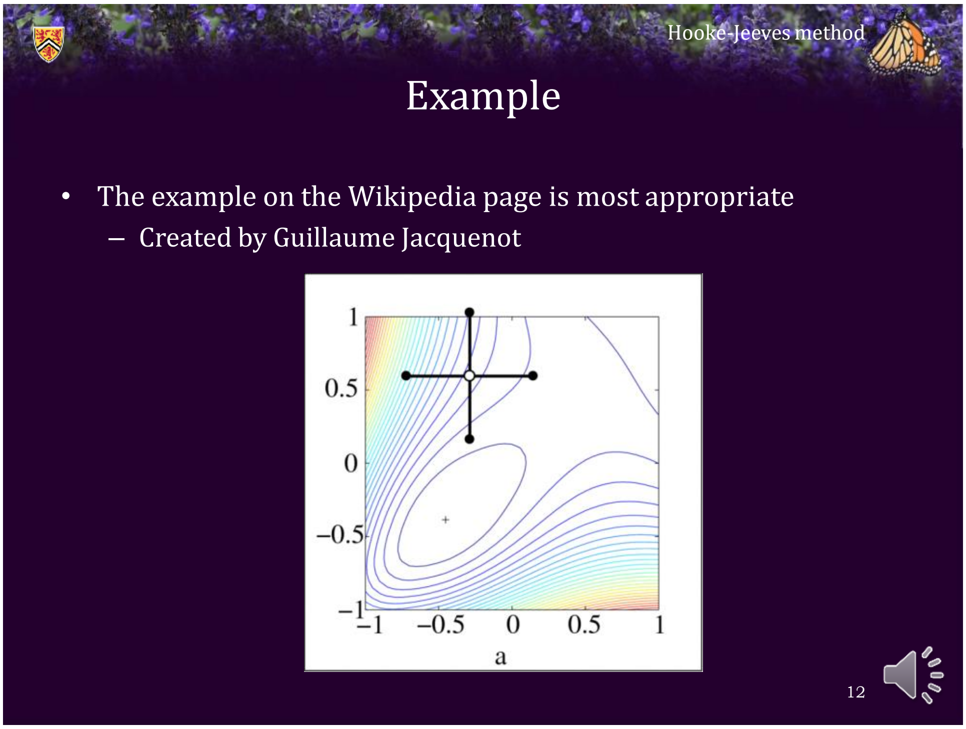
Implementation

```
// Check conditions
if ( norm( du ) == 0.0 ) {
    if ( h < eps_step ) {
        return std::make_pair( u, min );
    } else {
        h /= 2.0;
        continue;
    }
}
```



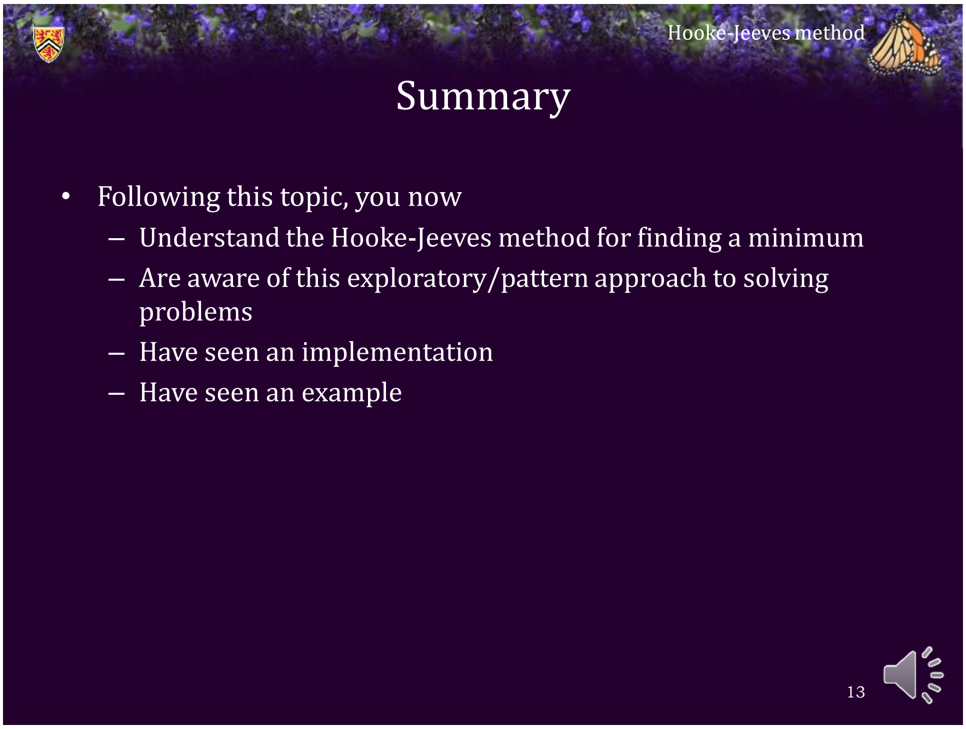
Implementation

```
// Pattern moves
//  - We stored the initial values in u0 and min0

while ( k < max_iterations ) {
    double fm{ f( u + du ) };
    ++k;

    if ( fm < min ) {
        u += du;
        min = fm;
    } else {
        break;
    }
}

if ( (k < max_iterations) && (norm( u - u0 ) < eps_step)
                          && ((min0 - min) < eps_abs) ) {
    return std::make_pair( u, min );
}
```



Example

- The example on the Wikipedia page is most appropriate
  - Created by Guillaume Jacquenot

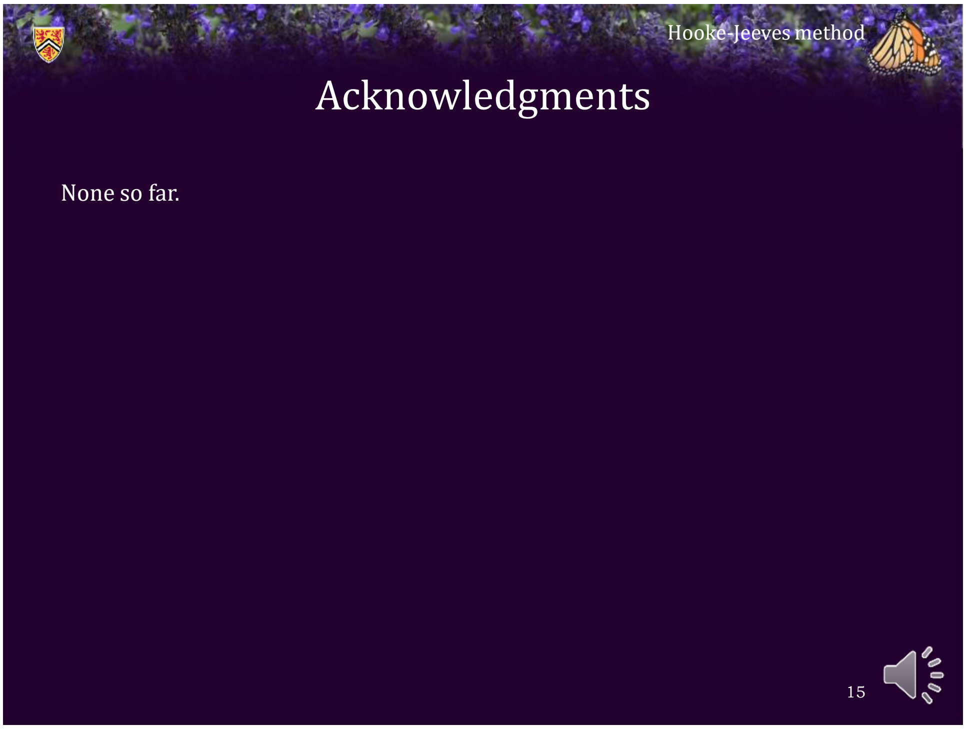


Summary

- Following this topic, you now
  - Understand the Hooke-Jeeves method for finding a minimum
  - Are aware of this exploratory/pattern approach to solving problems
  - Have seen an implementation
  - Have seen an example

References

[1] https://en.wikipedia.org/wiki/Pattern_search_(optimization)



Hooke-Jeeves method

Acknowledgments

None so far.

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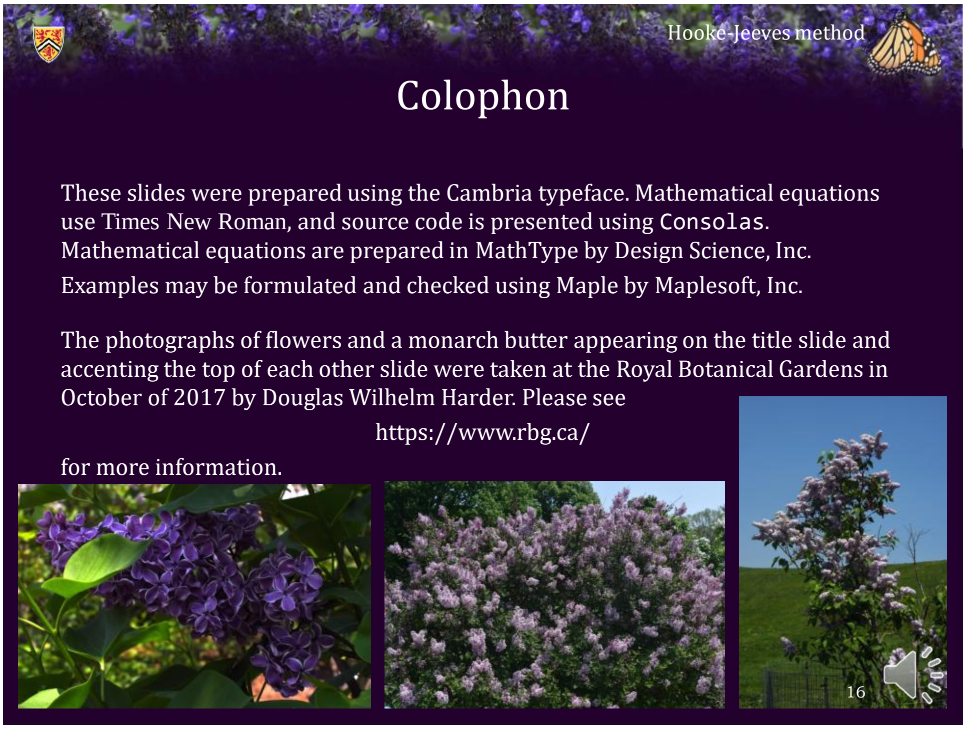

Hooke-Jeeves method

Colophon

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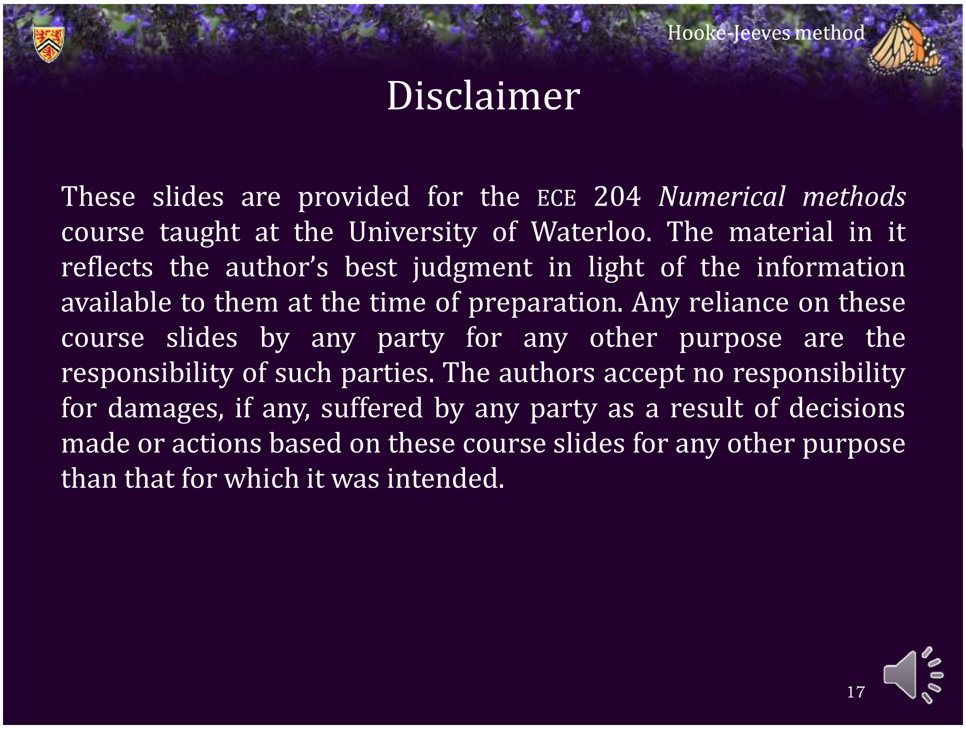

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